REPORT No. 123

SIMPLIFIED THEORY OF THE MAGNETO

By F. B. SILSBEE
Bureau of Standards

REPORT No. 123

SIMPLIFIED THEORY OF THE MAGNETO.

By F. B. Silsbee.

RÉSUMÉ.

This paper contains part of the results of ignition investigations being made for the National Advisory Committee for Aeronautics at the Bureau of Standards, and describes a type of circuit which has been found useful for representing the action of the high-tension magneto. While this equivalent circuit is relatively simple, and consequently can be used as a basis for deriving definite mathematical formulas for induced voltages and similar quantities, it has been found experimentally to correspond quite closely in its performance with the highly complicated electrical circuits of an actual magneto. In the paper formulas are given for the voltage induced in the secondary under various conditions of operation, and a number of numerical examples are worked out showing the application of the equations to a variety of practical problems.

INTRODUCTION.

The high-tension magneto is an exceedingly complex piece of apparatus and even the most detailed mathematical treatments as yet attempted can take account of only the more important features of its operation and are necessarily based upon numerous limiting assumptions.[1] This fact, together with the lack of familiarity of many automotive engineers with electrical concepts, has contributed to produce a general air of mystery regarding the operation of ignition apparatus and has permitted the growth of numerous fallacies and the exploitation by misleading claims of devices for remedying ignition troubles. There is consequently need for a fuller understanding of the principal characteristics of magnetos by engineers and even by manufacturers of ignition apparatus.

A very rough approximation is, however, for most purposes all that is required, since the device is always used in connection with spark gaps which are themselves subject to very great variations in breakdown voltage from a number of conditions, only a few of which are at present thoroughly understood. It is therefore possible by suitable simplification and the elimination of unnecessary elements to develop a model electric circuit which will represent all the important features of operation of a magneto in a qualitative manner and do this with an accuracy which is sufficiently good for much quantitative work. Such an abstract theoretical circuit, provided it exhibits the same principal characteristics as the actual machine, will be referred to as a "model" of the magneto.

It is the purpose of the present paper to develop the theory of such a model which, though appearing at first glance very remote from the actual device, has been found to give a fairly close approximation to the actual working of the magneto; to show the application of this model through the entire cycle of operations; and to give a few examples showing how it may be used to answer the various questions which may arise in practice as to the performance of ignition apparatus under various conditions. This theory is the result of the investigation of ignition apparatus, sponsored by the National Advisory Committee for Aeronautics, which has been carried on at the Bureau of Standards for the past several years and has proved very useful in correlating the experimental results obtained in the course of the work.

[1] Young, A. P., Automobile Engineer 7, pp. 191, 237, 262, 298, 1917; Taylor-Jones, E., Phil. Mag. 36, p. 145, August, 1918; Silsbee, F. B., Scientific Paper of the Bureau of Standards, "The Mathematical Theory of the Induced Voltage in the High-Tension Magneto."

DEVELOPMENT OF MODEL.

The high-tension magneto serves two distinct functions. First, as an electric generator it transforms mechanical energy supplied to it by the driving shaft into stored electromagnetic energy of the primary current flowing in the inductive primary circuit. This action is performed by the primary coil alone and is similar to the behavior of any type of generator and hence need not be discussed in much detail. The second, and less understood function, is that of an induction coil which by a sudden effort delivers the stored energy at a voltage sufficiently high to break down the spark gap in the engine cylinder and produce a spark.

This latter action is analogous to that of a motor-truck driver endeavoring to take his vehicle up a curbing or to jerk it out of a crust of frozen mud in which it has stood over night, and involves two distinct actions. In the first place, by speeding up his engine and letting in the clutch suddenly, the driver makes use of the inertia of the flywheel to exert on the clutch shaft a much greater torque than the engine could deliver continuously. Secondly, the multiplying action of the transmission gears delivers at the driving axle a torque which has been still further increased in proportion to the gear ratio. In a similar manner, by establishing a considerable primary current (speed) and suddenly interrupting it at the breaker, there is induced in the primary circuit (clutch shaft) a voltage (torque) much larger than that originally generated by the rotation of the armature. This voltage is then in effect multiplied in the secondary winding (axle) by the ratio of turns between the two windings.

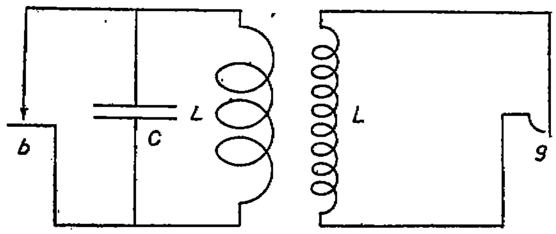

FIG. 1.—Circuits of actual magneto.

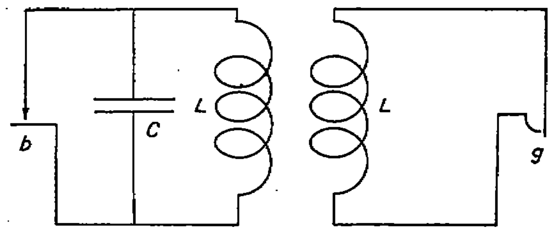

FIG. 2.—First intermediate type of circuit with secondary constants reduced to primary side.

Now the increase in voltage due to the great number of secondary turns is primarily a multiplication by the ratio (n) of secondary to primary turns, and we may consequently make the first simplifying step by replacing the circuits of figure 1 by those of figure 2 in which the secondary has been replaced with a fictitious winding having the same number of turns as the primary but occupying the same space as the actual secondary winding. Any values of voltage computed for this type of circuit may be accurately converted to those corresponding to the actual machine by multiplying by (n). It may be shown that the effect of the turn ratio upon current is the reverse of that upon voltage, so that the secondary current in the actual machine is $1/n$th of that in the model shown in figure 2. This relationship is of course parallel to the inverse relation between torque and speed in cases where energy is transmitted through gearing. Furthermore, in the electrical case, it may be shown that any resistance or inductance in the secondary circuit of figure 1 should be replaced in figure 2 by one having $1/n^2$ times this value, while any electrostatic capacity should be replaced by one of n^2 times the value. This transformation from the actual circuits of figure 1 to the fictitious circuit of figure 2 is widely used in electrical engineering problems involving transformers or telephone repeating coils. This transformation may be extended to any apparatus connected to the secondary circuit, as well as to the parts of the secondary winding itself, since a group of circuits whose constants have been modified as outlined above will obey the same electromagnetic laws (Ohm's law, for example) as did the unmodified circuit.

The next step in the simplification in figure 2 is to note that the two coils which now have the same number of turns are wound upon a common iron core and that by far the greater part of any magnetic flux which may be produced by the action of one coil must necessarily link the turns of the other coil. Experiments have shown that the coefficient of coupling k, which is defined as the ratio of the mutual inductance between the two coils to the geometric mean of their respective self-inductances, is usually as great as 0.98 in the case of magnetos and 0.95 in the case of ignition coils in which the iron circuit is not as completely closed. In either case the amount of energy associated with the leakage flux which does not link both coils is very small, and we may therefore make the approximation that the coils are strictly closely coupled as is indicated by figure 3.

The next step is to replace the two closely coupled coils of figure 3, which, by our hypothesis, are identical in all their magnetic effects, by the single coil of the same number of turns and the

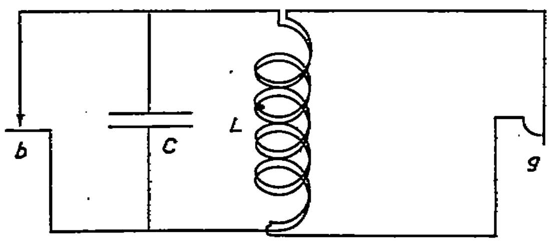

FIG. 3.—Second intermediate type of circuit with close magnetic coupling between primary and secondary.

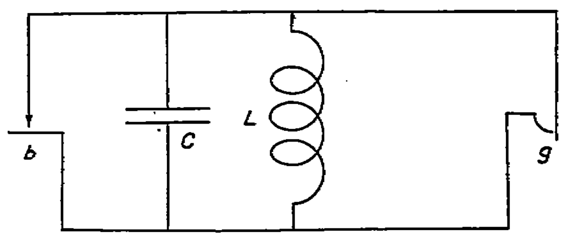

FIG. 4.—Third intermediate type of circuit with direct coupling between primary and secondary.

same self-inductance shown in figure 4. The only physical change which has been made in this step is that we no longer have the two coils separated by an insulating barrier, but since this insulation does not directly affect the calculation of voltages, currents, etc., it need not concern us here.

In any device for transforming energy there is, of course, always a certain amount of loss which, in the case of a magneto, is partly due to the resistance of the copper winding and in part to the hysteresis and eddy current loss in the iron core. We must, therefore, so arrange our model as to account for such losses. This can be done very easily by inserting the resistance R in series with the coil. (See fig. 5.) It should be noted, however, that when we are concerned with relatively slow changes in current and voltage, as for example during the building up of current by the rotation of the armature, only the copper resistances are of importance, while when very rapid changes are in progress, as during the building up of voltage immediately after the primary contact points have opened, the eddy current loss in the core produced by the rapid change of flux becomes much larger than the energy dissipation in the copper winding.

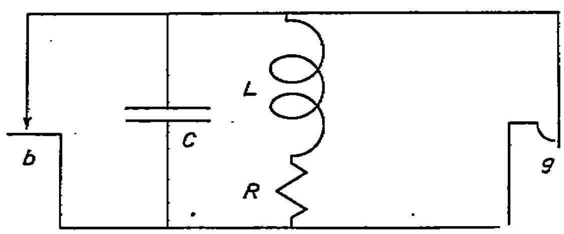

FIG. 5.—Complete single-coil model with series resistance.

Consequently, the numerical value to be assigned to the resistance R should be taken at different values according to the part of the cycle of operation under consideration.

A detailed discussion of methods for the determination of numerical values of the resistance in these cases and also of the inductance and capacity, if the model is to represent any given magneto, is beyond the scope of the present paper, as such methods have been treated elsewhere.[3]

In the paper referred to several lines of attack are suggested, the most direct of which is the measurement, by means of the Wheatstone bridge supplied with alternating current, of the inductance and resistance of the magneto winding at a frequency approximately the same as

[3] See the latter part of Scientific Paper of Bureau of Standards, "The Mathematical Theory of the Induced Voltage in the High-Tension Magneto." This paper also gives a more detailed discussion of the single-coil model and compares results obtained by it with those computed from other types of model and with the results of direct experiment.

that of the oscillations which would occur in the model. The proper value of the capacity to use is obtained by combining the easily measured value of the primary condenser with a value of the secondary capacity (multiplied of course by n^2) obtained by measurements made at radio frequencies. The ratio of turns of the magneto can readily be obtained by a null method, such as described in National Advisory Committee for Aeronautics Report No. 58, Part II. For most qualitative applications of the model, a precise knowledge of the values of the circuit constants is not necessary and, as will be indicated below, the equations which will be developed enable one to apply the model to numerous quantitative problems without requiring a complete knowledge of the constants.

FORMULAS AND APPLICATIONS OF MODELS.

During the first part of the cycle of operations of the magneto, when the rotation of the armature is building up current in the primary winding through the closed breaker contacts, the simplified model is of relatively little assistance. The wave form of the E. M. F. induced by the rotation is quite complex and depends entirely upon the shape of armature, pole pieces, and other parts of the magnetic circuit. Certain general considerations may, however, be drawn from the model which during this entire period (called "Period 1") may be considered as consisting merely of the portion shown in figure 6, since the breaker contacts form an effective short circuit upon the condenser and prevent it from affecting the operation.

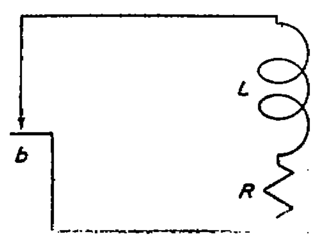

FIG. 6.—Effective circuit of model during Period 1 while current is being generated.

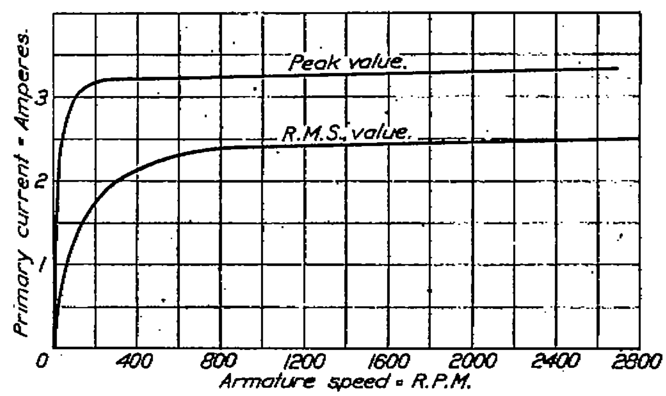

FIG. 7.—Data showing constancy of primary current of magneto after critical speed is attained.

The circuit as thus reduced consists of a resistance in series with an inductance supplied by an alternating E. M. F. This E. M. F. is proportional to the speed of the armature, the number of turns in the primary winding, and the total magnetic flux. If the voltage were sinusoidal, the current would consequently be given by the equation

$$I=\frac{E}{\sqrt{R^2+(2\pi fL)^2}}=\frac{K\,n_p f}{\sqrt{R^2+(2\pi fL)^2}} \tag{1}$$

where R is the resistance, n_p the number of turns, and L the inductance of the primary coil, and f is the frequency (or in a 2-pole machine the revolutions per second). If the speed of rotation is small, the term in the denominator involving the inductance is small relative to the resistance, and we note that the current will be proportional to the voltage and hence to the speed. At higher speeds, however, the inductance term becomes preponderant, and since it increases in proportion to the frequency, the current becomes independent of the frequency. As an example of this, figure 7 shows the effective (root-mean-square) current on short circuit and also the peak values of the same current observed in a certain magneto at several armature speeds. The effect of the inductance is greater upon any overtones which may be present in the current wave form, and consequently the wave shape changes considerably with frequency and the peak value of the current becomes independent of speed at a somewhat lower value than does the effective value. As a critical speed, above which the current at break and hence the amount of energy available for the rest of the operation may be deemed substantially constant in value, we may take that defined by the equation

$$S \text{ (in r. p. m.)} = \frac{60}{2\pi} \frac{R \text{ (in ohms)}}{L \text{ (in henrys)}} \text{ approx.} \tag{2}$$

The energy which is stored in the magnetic field as a result of the current thus established is given by equation

$$W = \tfrac{1}{2} LI^2 \tag{3}$$

In some cases when the cam is set to open the contacts early in the rotation an additional amount of energy may be supplied to the spark by the rotation of the armature after the spark gap has broken down and provided a conducting path between its electrodes. This additional energy, however, does not contribute to the production of voltage to break down the gap except to the extent that the E. M. F. of rotation of the secondary winding in the magnetic field is added to the E. M. F. induced by the interruption of the primary current.

After the breaker contacts have been separated by the cam, we have an entirely distinct set of conditions which exists for a very short interval which may be called "Period 2," which lasts until the spark gap breaks down. Although this interval is very short, and there is usually but little transformation of energy during it, the phenomena which occur are extremely important

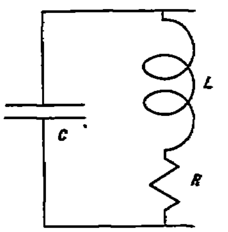

FIG. 8.—Effective circuit of model during Period 2 after breaker has opened and while voltage is building up.

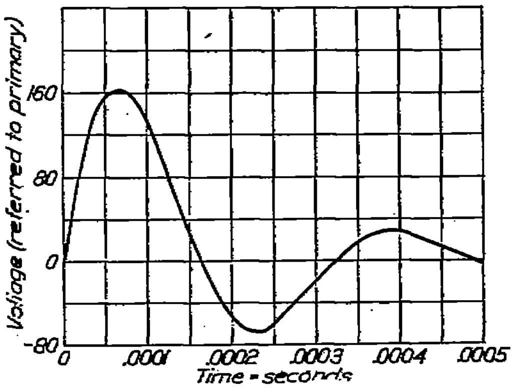

FIG. 9.—Voltage oscillation as computed for single coil model having L=.012 h; C=.215 μ f; R=127 ohms: assuming that the spark gap does not break down.

since they determine the amount of voltage which is produced in the secondary and, consequently, whether or not any spark at all is produced. The simplified model is developed especially to represent the magneto during this period and the portions of the model which are effective are shown in figure 8, in which it will be noted that the breaker circuit does not enter into the process. If such a circuit were left to itself undisturbed, the voltage across the condenser would rise to a maximum, at which time the current in the coil would have fallen to zero and would then decrease again oscillating thus until all the energy had been dissipated by the resistance. Figure 9 shows the variation of voltage with time in such a hypothetical case and is computed in a model where $L = 0.012\ h$, $C = 0.215\mu f$, $R = 127\Omega$. The complete equation for the voltage in such a case is

$$v = \frac{I_0}{\beta C} \epsilon^{\alpha t} \sin \beta t \text{ .... (volts)} \tag{4}$$

where

I_0 = current at break (amperes).

$$\alpha = \frac{-R}{2L} \text{ (seconds}^{-1}\text{).} \tag{5}$$

$$\beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ (seconds}^{-1}\text{).} \tag{6}$$

t = time (seconds).

e = Napierian base = 2.718

In most problems, however, we are interested merely in the value of the first maximum, since no spark will pass unless this maximum value is greater than the breakdown voltage of the gap at the spark plug. The value of this maximum is given by the equation

$$V_m = I_0\sqrt{\frac{L}{C}} \times F \text{ (volts)} \quad (7)$$

TABLE I.—*Damping factor F (for use in equation 7).*

$$F = e^{\frac{\alpha}{\beta} \tan^{-1} \frac{-\beta}{\alpha}}$$

$\frac{\alpha}{\beta}$	F
—0.0	1.000
.1	.863
.2	.760
.3	.681
.4	.621
.5	.575
.6	.539
.7	.511
.8	.488
.9	.470
1.0	.456

TABLE II.—*Damping factor F′ (for use in equation 12).*

$$F' = \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\frac{\alpha_1}{\alpha_2}}{1-\frac{\alpha_1}{\alpha_2}}}$$

$\frac{\alpha_1}{\alpha_2}$	F'
0.0	1.000
.1	.774
.2	.669
.3	.597
.4	.543
.5	.500
.6	.465
.7	.435
.8	.410
.9	.387
1.0	.368

The factor F in equation (7) is an abbreviation for the expression

$$F = e^{\frac{\alpha}{\beta} \tan^{-1} \frac{-\beta}{\alpha}} \quad (8)$$

Table (I) gives values of this factor for various values of the ratio $\frac{\alpha}{\beta}$

In certain cases where the rate of dissipation of energy is very great, the voltage wave no longer oscillates, but having risen to a maximum subsides gradually to zero. Under these conditions, the equations (4) and (7) no longer apply, and the equation expressing the voltage as a function of time becomes

$$v = \frac{I_0}{C(\alpha_1 - \alpha_2)}\left[e^{\alpha_1 t} - e^{\alpha_2 t}\right] \text{ (volts)} \quad (9)$$

where

$$\alpha_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \text{ (seconds}^{-1}\text{)} \quad (10)$$

$$\alpha_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \text{ (seconds}^{-1}\text{)} \quad (11)$$

While the maximum voltage of the impulse is given by equation

$$V_m = \frac{-I_0}{C\alpha_2} \times F' \text{ .... (volts)} \quad (12)$$

The factor F' in this equation is analogous to the factor F in the preceding case and is an abbreviation for the expression

$$F' = \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\frac{\alpha_1}{\alpha_2}}{1-\frac{\alpha_1}{\alpha_2}}} \quad (13)$$

Table (II) gives values of this factor for various values of the ratio $\frac{\alpha_1}{\alpha_2}$

In normal operation the circuits are usually such as to give the oscillatory case first considered, and the factor F has approximately the value 0.75, although this depends to a considerable extent upon the construction, in particular upon the amount of lamination of the magnetic circuit.

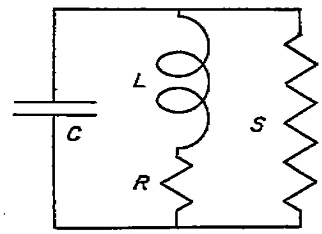

FIG. 10.—Effective circuit of model during Period 2 if spark plug is fouled with carbon.

In case the magneto secondary is shunted by a resistance, as is the case when a spark plug is fouled with carbon deposits, we have the somewhat more complicated circuit shown in figure 10. This can, however, be simplified to the circuit shown in figure 8 by making the following substitutions:

$$\begin{aligned} L' &= L, \text{ (henrys)} \quad C' = C \times \frac{S}{R+S} \quad \text{(farads)} \\ R' &= R + \frac{L}{SC} \text{ (ohms)} \quad I_0' = I_0 \quad \text{(amperes).} \end{aligned} \quad (14)$$

In most cases this substitution has relatively little effect upon the value of capacity, but the term $\frac{L}{SC}$ in equation (14) which results from a moderately low value of the shunting resistance S in the circuit of figure 10 is so great that the original resistance R in the unshunted combination contributes practically a negligible amount to the damping. Under these circumstances, the maximum voltage is given by the equation

$$V_m' = \frac{I_0\sqrt{LC'}}{C} F \text{ .... (volts)} \quad (15)$$

or in case the oscillation is overdamped, by

$$V_m' = \frac{I_0'}{C\alpha_2'} F' \text{ .... (volts)} \quad (16)$$

in which should be noted that the value of C and not C' is to be used in the denominator. The factor F or F', however, is the same as occurs in equation (8 or 13) above, provided the primed values obtained by the substitutions (14) are used.

It may be noted that the presence of a shunt, as shown in figure 10, gives rise to effects very similar to those due to eddy currents in the iron core, and the substitutions defined by equation (14) indicate the reason for the effect of these currents in increasing the effective series resistance of the circuit of figure 8.

When the shunting is very heavy, as is usually the case when the magneto is loaded nearly to the point of misfiring, equation (16) reduces to the much more simple form

$$V_m = I_o S \tag{17}$$

The physical reason for this may be seen by the fact that if the full current I_o should flow through the shunting resistance S, it would maintain the initial magnetism in the coil unchanged. With such a constant flux no voltage would be induced to maintain the current flow. Consequently this initial value constitutes an upper limit to the current which will flow through such a shunting resistance and will, of course, produce at the terminals of the resistance the voltage given by equation (17).

While the oscillations and maximum voltages indicated by the preceding equations would be produced if the circuits in figure 8 (or 10) were left undisturbed, in the normal operation of an ignition system the breakdown voltage of the spark plugs is normally adjusted to a value much smaller than the maximum attainable by the system so as to insure firing by a factor of safety of 4 or 5. As soon as the spark gap breaks down, it provides a conducting path in parallel

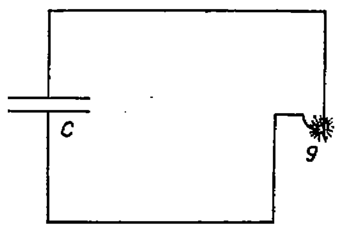

FIG. 11.—Effective circuit of model during Period 3 when spark gap has just broken down.

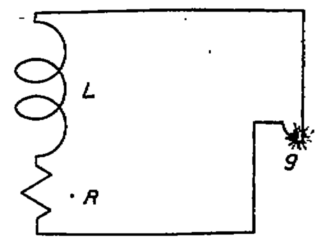

FIG. 12.—Effective circuit of model during Period 4 after the capacities have discharged but while the coil discharge maintains the spark.

with the model and completely changes the electrical system so that from this instant on we have to deal with the circuit shown in figure 11. The principal action which first occurs (and which may be called "Period 3") is the discharge of the condenser through the spark gap, and because of the very small resistance and inductance of the leads and of the spark itself, the resulting current may attain very considerable values for a short interval of time; consequently the slight contribution to the total current from the inductance itself is negligible and the coil has been omitted from figure 11.

In the actual magneto, the slight amount of leakage flux which is present because the two windings are not closely coupled becomes of some importance in the very rapid discharge which takes place during this Period 3, and consequently the first rush of current comes only from that part of the condenser due to the secondary leads and winding. The contribution from the primary condenser follows, however, very shortly afterward, and the entire phenomenon may be considered as a unit. The energy delivered to the spark gap during this period is given by

$$W = \tfrac{1}{2} CV^2 \tag{18}$$

where V is the voltage at which the gap broke down. This energy is usually only a small fraction of the original magnetic energy stored in the coil, but is probably sufficient to produce ignition of the gas mixture in the engine cylinder.

After the condensers have discharged we are left with the spark gap still in a conducting condition, and consequently have the circuit shown in figure 12 during what may be called "Period 4." Such a coil left to itself would tend to send a current nearly equal to that originally flowing through the breaker circuit, since such a current would be sufficient to maintain the

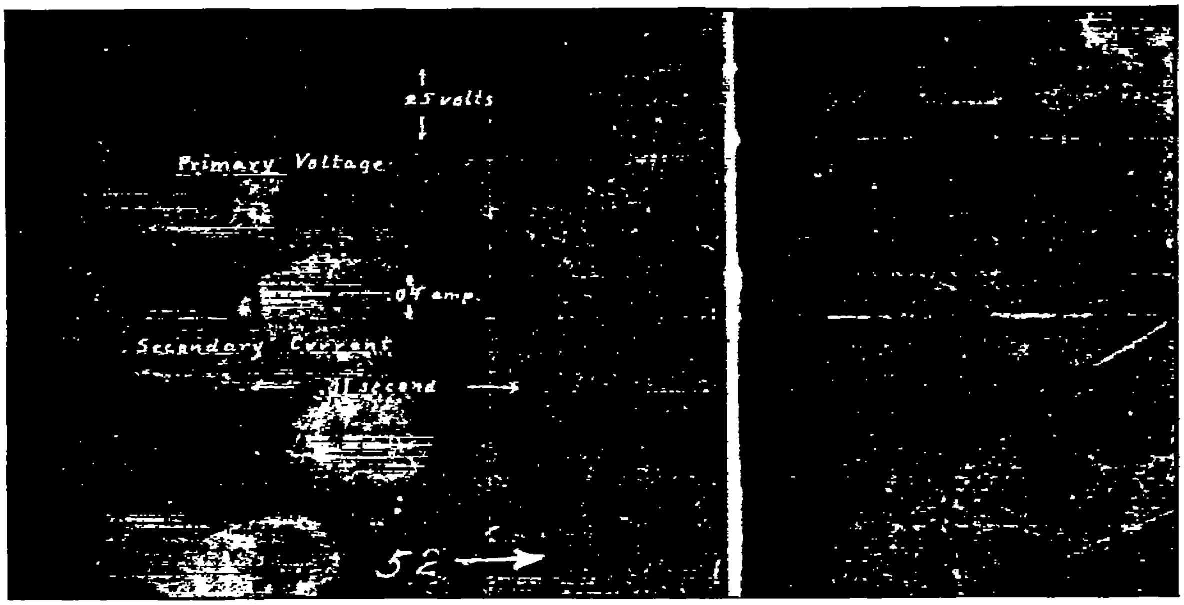

FIG. 13.—OSCILLOGRAM OF SLEEVE TYPE MAGNETO SHOWING IN UPPER RECORD THE NEARLY CONSTANT VOLTAGE SUSTAINING THE SPARK, AND THE LOWER RECORD THE NEARLY LINEAR DECREASE OF CURRENT WITH TIME DURING PERIOD 4.

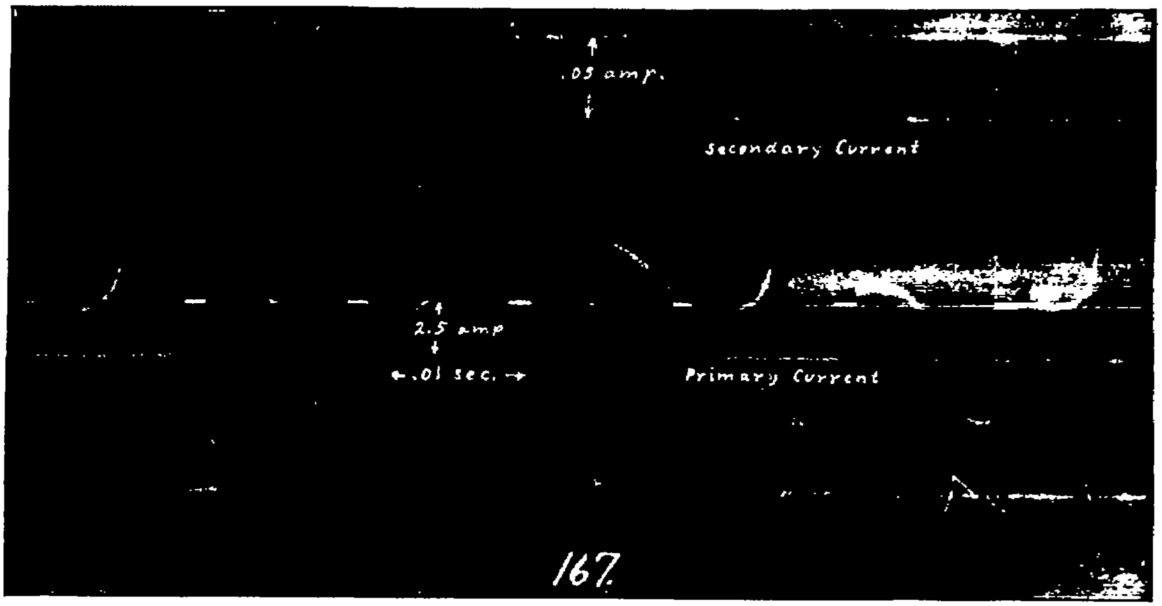

FIG. 15.—OSCILLOGRAMS OF SECONDARY (UPPER) AND PRIMARY (LOWER) CURRENT OF SHUTTLE TYPE MAGNETO. NOTE THE CORRESPONDENCE BETWEEN THE DISTURBANCES IN THE PRIMARY CURRENT CAUSED BY THE MECHANICAL "CHATTERING" OF THE BREAKER AND THE CORRESPONDING FLUCTUATIONS IN THE SPARK CURRENT. PLOTTED TO THE SAME SCALE OF AMPERE TURNS THE PEAKS OF ONE CURVE FIT INTO THE HOLLOWS OF THE OTHER.

magnetic flux undiminished. The actual current, therefore, closely approximates that which was present at the end of Period 2 and is roughly the same as that existing at the end of Period 1. It is a curious property of the type of spark gap used in engine cylinders that they require to maintain the current across them a voltage which is substantially constant during the duration of the spark. The value of this voltage depends upon the length of the gap and other conditions, and is of the order of magnitude of 1,000 volts. Since the flux is changing but slowly during the period now being considered, the effective resistance of the model is small and the IR drop in the winding is relatively slight. We consequently have roughly a constant rate of fall of current which thus maintains a constant voltage $\left(L\frac{di}{dt}\right)$ across the gap. This is illustrated by figure 13, which shows oscillograms of the secondary current through the spark gap and of the voltage across the primary coil terminals during the passage of the spark. Unless interrupted by the early closing of the breaker, this discharge of the coil continues until the current has dropped off uniformly to zero and the total energy delivered to the spark gap is given by

$$W=(EI)_{ave}t \tag{19}$$

This contribution is usually the bulk of the original magnetic energy stored in the coil, but in case the magneto has been working near its limit, so that the breakdown voltage of the spark gap was nearly equal to V_m, the energy will be definitely less than the original supply by the amount of electrostatic energy contributed from the condensers during Period 3.

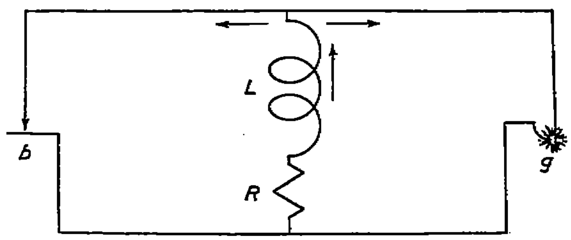

FIG. 14.—Effective circuit of model while spark is being extinguished by the closing of the breaker. (Period 5.)

In case the cam allows the primary breaker to close before the current through the spark gap has entirely died out, we get the condition shown in figure 14. On account of the changes in ionization it happens that, in the case of conduction across spark gaps, the resistance increases as the current flow through the gap decreases. Consequently, when the current begins to divide between the two parallel paths (b and g) shown in the figure, the resistance of path g rises as the current through it diminishes, with the result that the entire current is rapidly shifted to the circuit through the breaker, and the spark is extinguished. If for any reason the contacts bounce open again, as may occur at high speeds of operation, the current flow is again shifted to the spark gap, which is usually sufficiently ionized to remain conducting. Figure 15 shows this transfer of the current from primary to secondary, and vice versa, the upper curve in the record being that of the secondary current to a scale of 0.004 ampere per millimeter, while the lower curve gives the primary current to a scale of 0.3 ampere per millimeter. Since the ratio of turns in this particular magneto was 57:1, it will be seen that the ampere turns, and hence the current in the coil of the equivalent model, remains substantially constant in spite of the shift in the external path of the current.

EXAMPLES.

The following numerical examples may help to render the preceding discussion more definite and concrete and to illustrate the extent to which the model may be applied even in cases where a complete knowledge of the circuit constants is not available. In several cases the examples have been chosen to fit particular magnetos for which the answers have been verified experimentally.

EXAMPLE 1.

A certain magneto will barely fire a 6,100-volt spark gap when a current of 1 ampere passing through its primary is interrupted. If a condenser of 300 micro-microfarads capacity is connected to its secondary, it will then, with the same primary current, barely fire a 4,100-volt gap. What is its original effective capacity?

Let C_0 be the total original capacity (referred for convenience to the secondary side). Then the total energy available under the first condition is

$$W=\frac{(6,100)^2}{2}\cdot C_0$$

The energy in the second condition is

$$W=\frac{(4,100)^2}{2}\cdot(C_0+300\times10^{-12})$$

Since the same primary current was used in both cases, the initial energy is the same in both, and we may equate these two expressions, getting the relation

$$\frac{(6,100)^2}{2}C_0=\frac{(4,100)^2}{2}(C_0+300\times10^{-12})$$

which may be solved to give

$$C_0=246\times10^{-12}\text{ farads.}$$

EXAMPLE 2.

In the magneto of example 1, assuming that the damping factor F is 0.75 and that the ratio of turns is 60, what is the effective inductance and resistance?

The current at break referred to the secondary is $\frac{1}{60}$ amperes and by equation (7) we have

$$6,100=\frac{1}{60}\sqrt{\frac{L}{246.10^{-12}}}\cdot 0.75$$

whence

$$L=59\text{ henrys.}$$

Now a factor F of 0.75 corresponds to $\frac{\alpha}{\beta}=-0.2$ approximately and

$$\beta=\frac{1}{\sqrt{LC}}\text{ approx.}=\frac{1}{\sqrt{59\times246\ .10^{-12}}}=8,330.$$

The corresponding frequency is

$$f=\frac{8,330}{2\pi}=1,330$$

and

$$\alpha=-0.2\times8,330=-1,670.$$

hence

$$R=-2\ L\ \alpha\text{ by equation (5)}$$

$$=2\times59\times1,670=197,000\text{ ohms.}$$

Referred to the primary side, the various quantities would be

Current at break		$=1.0$ amperes.
Crest voltage	$=\frac{6,100}{60}$	$=102$ volts.
Resistance (for period 2)	$=\frac{197,000}{(60)^2}$	$=54.7$ ohms.
Inductance	$=\frac{59}{(60)^2}$	$=.0164$ henrys.
Capacity	$=246\times10^{-12}\times(60)^2=.886\times10^{-6}$ farad.	

If the primary condenser has 0.25 microfarad capacity what is the secondary capacity?

Secondary capacity (considering the equations of the model to be referring to primary quantities) is equal to

$$0.886\times 10^{-6}-0.25\times 10^{-6}=0.636\times 10^{-6} \text{ farads.}$$

Referring this to the secondary again gives

$$\frac{0.636\times 10^{-6}}{(60)^2}=177\times 10^{-12} \text{ farads.}$$

EXAMPLE 3.

If the magneto referred to above is used with leads 2 meters long of a type of high voltage cable which has an electrostatic capacity of 300 micro-microfarads per meter, by how much is the crest voltage reduced?

As in example 1 we may equate the electrostatic energies in the two cases and write

$$\frac{1}{2}\cdot\ (6{,}100)^2\times 246\times 10^{-12}=\frac{1}{2}\,(V_m)^2\times(246\times 10^{-12}+2\times 300\times 10^{-12})$$

which gives $V_m=3{,}300$, or a decrease of

$$\frac{6{,}100-3{,}300}{6{,}100}=46 \text{ per cent.}$$

(NOTE.—The decrease observed experimentally on the magneto which gave the values taken in example 1 was 40 per cent when 600 $\mu\mu$ were added.)

EXAMPLE 4.

If an accumulation of dirt in the distributor amounts to a shunting resistance of 500,000 ohms from the secondary terminal to ground, by how much is the crest voltage reduced?

We may refer all quantities to the secondary and then make the substitutions indicated by equation (14) obtaining

$$L'=L=59 \text{ henrys}$$

$$S=500{,}000 \text{ ohms}$$

$$R'=R+\frac{L}{SC}=197{,}000+\frac{59}{500{,}000\times 246\times 10^{-12}}=677{,}000 \text{ ohms}$$

$$C'=\frac{CS}{R+S}=\frac{246\times 10^{-12}\times 500{,}000}{197{,}000+500{,}000}=176\times 10^{-12}$$

Inserting these in equations (5 and 6) gives

$$\alpha'=\frac{-R'}{2\,L'}=-5{,}740$$

$$\beta'=\sqrt{96.\times 10^6-33.\times 10^6}=7{,}940$$

since α'^2 is less than $\frac{1}{LC'}$ the complete wave form is oscillatory and equation (7) applies.

$$\frac{\alpha'}{\beta'}=-0.723, \text{ hence } F=.506 \text{ by Table (I)}$$

and by equation (15)

$$V_m=\frac{I_o\sqrt{59\times 176\times 10^{-12}}}{246\times 10^{-12}}\times 0.506=210{,}000\ I_o$$

For 1 ampere in the primary, I_o referred to the secondary is $\frac{1}{60}$ and hence

$$V_m = \frac{210,000}{60} = 3,500 \text{ volts}$$

Decrease resulting from shunting is

$$\frac{6,100 - 3,500}{6,100} = 43 \text{ per cent.}$$

(NOTE.—The decrease observed experimentally on this magneto was 48 per cent.)

EXAMPLE 5.

If this magneto is connected to a spark plug which has the gap set to breakdown at 2,000 volts, what is the lowest value (S) of the resistance of a carbon deposit on the spark-plug insulator at which the magneto will fire the plug, if its primary current at break is 4 amperes?

For this case the approximate equation (17) is applicable. The current at break referred to the secondary is $\frac{4}{60} = 0.067$ ampere, and we therefore can write

$$2,000 = S \times 0.067$$

whence

$$S = 30,000 \text{ ohms.}$$

EXAMPLE 6.

If the primary condenser could be reduced to 0.15 microfarad without causing excessive arcing, what would be the gain in crest voltage?

The present capacity referred to the primary is (see example 2) 0.886 microfarad.

The present crest voltage referred to primary is 102 volts.

New capacity will be 0.786 microfarad.

Since total energy is the same in the two cases, we have

$$\frac{1}{2}(102)^2 \times 0.886 \times 10^{-6} = \frac{1}{2}(V_m)^2 \times .786 \times 10^{-6}$$

whence

$$V_m = 108$$

$$\text{the gain} = \frac{108 - 102}{102.} = 5.9 \text{ per cent.}$$

(NOTE.—The gain observed experimentally on the secondary was 5.9 per cent.)

EXAMPLE 7.

If the magneto operating with a primary current at break of 4 amperes is connected to a clean spark plug which has its gap set to break down at 2,000 volts: (a) What is the energy in the condenser part of the spark? (b) Neglecting losses, what is the total energy in the spark?

The condenser energy by equation (18)

$$W = \frac{1}{2} \,.(2000)^2 \times 246 \times 10^{-12} = .0005 \text{ joule.}$$

The total energy is by equation (3)

$$W = \frac{1}{2} \,.0.0164 \times (4)^2 = 0.131 \text{ joule.}$$

EXAMPLE 8.

Assuming that a constant voltage of 800 volts is required to maintain the spark at the spark plug, how long would the spark last if it were not interfered with by the closing of the primary breaker?

The initial current in the spark (during Period 4) is approximately

$$I = \frac{4}{60} = 0.067 \text{ ampere.}$$

The final current is zero and the current decreases uniformly.

The average current is therefore

$$\frac{0.067+0}{2} = 0.033 \text{ ampere.}$$

Now the energy (assuming no losses) is equal to

$$W = E\,I\,t = 800 \times 0.033 \times t = 0.131 - 0.0005$$

whence

$$t = 0.0049 \text{ second.}$$

EXAMPLE 9.

If the cam holds the breaker open during a rotation of 60°, at what speed will the closing of the breaker begin to interfere with the spark?

The time of opening is $\frac{60^\circ}{360^\circ}$ or one-sixth of a revolution, and if this is to be 0.0049 second, one revolution must occupy $6 \times .0049 = 0.03$ second, corresponding to 2,000 revolutions per minute.

NOTATION.

C = electrostatic capacity.
E = effective (root-mean-square) voltage.
ϵ = base of Napierian logarithms = 2.718 . . . .
f = frequency (cycles per second).
F, F' = correction factors for damping.
I = effective (root-mean-square) current.
I_o = current at instant of break.
K = any constant.
L = self-inductance.
n_p = number of turns of primary winding.
R = resistance.
s = speed of rotation of magneto armature.
t = time.
v = voltage at any instant.
V_m = maximum voltage.
W = energy.

$$\alpha = \frac{-R}{2L}$$

$$\alpha_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$\alpha_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$\beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$